

Exact solutions of a two-dimensional Duffin-Kemmer-Petiau oscillator subject to a coulomb potential in the gravitational field of cosmic string

Abdelmalek Boumali* and Nadjette Messai†

Laboratoire de Physique Appliquée et Théorique,

University Larbi Tébessi -Tébessa-, 12000, W. Tébessa, Algeria.

Abstract

In this paper, the problem of a two-dimensional Duffin-Petiau-Kemmer (DKP) oscillator in the presence of a coulomb potential in the cosmic string background is solved. The eigensolutions of the problem in question have been found, and the influence of the Coulomb potential in the presence of the gravitational field of cosmic string has been analyzed.

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*Electronic address: boumali.abdelmalek@gmail.com

†Electronic address: nadjette.messai@gmail.com

I. INTRODUCTION

The analysis of gravitational interactions with a quantum mechanical system has recently attracted attention in particle physics and has been an active field of research. The general way to understand the interaction between relativistic quantum mechanical particles and gravity is to solve the general relativistic form of their wave equations. These solutions are valuable tools for examining and improving models and numerical methods for solving complicated physical problems.

In the conventional relativistic approach, the interaction of $S = 0$ and $S = 1$ hadrons with different nuclei has been described by the second-order Klein-Gordon (KG) equation for $S=0$ and Proca equation for $S=1$ particles. It is well known that is very difficult to tackle these second-order equations mathematically and to derive the physics behind them. Therefore, considerable interest in recent years has been devoted to examining the interactions of $S = 0$ and $S = 1$ hadrons with nuclei by using the first-order relativistic Duffin-Kemmer-Petiau (DKP) equation [1].

One important question related to DKP equation concerns the equivalence between its spin 0 and 1 sectors and the theories based on the second-order KG and Proca equations[2]. Historically, the loss of interest in the DKP stems from the equivalence of the DKP approach to the Klein-Gordon (KG) and Proca descriptions in on-shell situations, in addition to the greater algebraic complexity of the DKP formulation. However, in the 1970s, this supposed equivalence was questioned in several situations involving breaking of symmetries and hadronic processes, showing that in some cases, the DKP and KG theories can give different results. Moreover, the DKP equation appears to be richer than the KG equation if the interactions are introduced. In this context, alternative DKP-based models were proposed for the study of meson-nucleus interactions, yielding a better adjustment to the experimental data when compared to the KG-based theory[3]. In the same direction, approximation techniques formerly developed in the context of nucleon-nucleus scattering were generalized, giving a good description for experimental data of meson-nucleus scattering[4]. The deuteron-nucleus scattering was also studied using DKP equation, motivated by the fact that this theory suggests a spin-1 structure from combining two spin- $\frac{1}{2}$ [5]. In addition, we can cite the works of [6, 7] on the meson-nuclear interaction and the relativistic model of α -nucleus elastic scattering where they have been treated by the formalism of the DKP

theory. Recently, there is a renewed interest in the DKP equation. It has been studied in the context of quantum chromodynamics (QCD) [8], covariant Hamiltonian formalism [9], in the causal approach[10, 11], in the context of five-dimensional Galilean invariance[12], in the scattering of K^+ nucleus[13], in the presence of the Aharonov-Bohm potential[14, 15], in the Dirac oscillator interaction[16], in the study of thermodynamics properties[17], on the supersymmetric[18], and finally in the presence of some shape of interactions[19–31]. These examples in some case break the equivalence between the theories based on the DKP equation and KG and Proca equations.

The Dirac oscillator was for the first time studied by Itô and Carriere [32]. On the other side, Moshinsky and Szczepaniak were the first who introduced an interesting term in the Dirac equation. More specifically, they suggested to substitute in the free Dirac equation the momentum operator \vec{p} like $\vec{p} - im\omega\beta\vec{r}$. They could obtain a system in which the positive energy states have a spectrum similar to the one of the non-relativistic harmonic oscillator [33]. Recently, this interaction has particularly got more interest. It is reviewed, because of the interest in the many different domain in physics (see Ref. [33] and references therein).

The topological defects plays an important role in physical properties of systems, and they appear in gravitation as monopoles, strings and walls [34–43]. Among them, cosmic strings and monopoles seem to be the best candidates to be observed. The former are linear defects, and the space-time produced by an idealized cosmic string is locally flat, however, globally conical, with a planar angle deficit determined by the string tension.

The well-known procedure to introduce the coupling between a charged particle and electromagnetic fields in the DKP equation, is through the minimal coupling. Dosch, Jensen and Müller in 1971 proposed another procedure by making a modification in the mass term in the form: $m \rightarrow m + S(\vec{r})$ where $S(\vec{r})$ is the scalar potential [45]. This new formalism has been used by Soff et al [46] to analyze the Dirac equation in the presence of a Coulomb potential and a static scalar potential. Bergerhoff and Soff [47] show that, in contrast to the minimal coupling of the electromagnetic potentials where it is correlated with the momentum, a scalar potentials, which are an invariant Lorentz's scalar, are coupled to the mass of a particle in the Dirac equation and thus act effectively as a position dependent mass. They have shown that, when a scalar external is coupled instead of a vector Coulomb potential, there is no present of Klein's paradox and consequently the spontaneous pair creation. This implies, that for an arbitrary scalar potential one can always find the bound states in the

gap between $+m_e c^2$ and $-m_e c^2$, which is not the case for the usual Coulomb potential, which is coupled to the Dirac field by replacing \vec{p} by $\vec{p} - (e/c)\vec{A}$ in the Dirac equation. More recently, Medeiros et al [44] have been used this formalism to study Relativistic quantum dynamics of a charged particle in cosmic string space-time in the presence of magnetic field and scalar potential. In the same context, Bakke and his co-workers [40] have been studied several problems by using this new approach. They showed that this modification in the mass term gives rise to a position-dependent mass for a relativistic particle (see Ref.[38] and references therein). Following Bakke [40], this method has been used in different situation such as : (i) the quark–antiquark interaction, (ii) analysis of the behavior of a Dirac particle in both static scalar and Coulomb potentials, (iii) in a relativistic scalar particle in the cosmic string space time, and (iv) finally in the Klein-Gordon oscillator subject to a Coulomb potential. Following Medeiros et al [44], if one wants to investigate the relativistic quantum motion of a charged particle in the presence of electromagnetic and scalar potentials, both procedures, the minimal coupling and a modification in the mass term, should be taken into account. The problem of the wave functions of particles subject to different confining potentials as a Coulomb potential, whose exact solution as well established, has been made by replacing \vec{p} by $\vec{p} - \frac{e}{c}\vec{A}$ in the relativistic particle equation's: this potential is a time-like component of the electromagnetic vector potential.

The principal aim of this paper is to solve the DKP oscillator in a background produced by topological defects, such as cosmic strings in the presence of a Coulomb potential. The introduction of the Coulomb potential in the DKP equation will be made by using the minimal coupling procedure. The structure of this article is as follows: In Sect. II, we briefly review the DKP equation in cosmic string background subject to the Coulomb potential. In Sect III, the eigensolutions have been obtained for both massive Spin-0 and spin-1 particles. Sect IV present our conclusion.

II. THE DKP OSCILLATOR IN COSMIC STRING BACKGROUND

In this section, we discuss the DKP oscillator in cosmic string space-time described by the metric [48, 49]

$$ds^2 = -dt^2 + d\rho^2 + \alpha^2 \rho^2 d\phi^2 + dz^2, \quad (1)$$

with $-\infty < (t, z) < +\infty$, $0 < \rho \leq \infty$ and $0 \leq \phi \leq 2\pi$. The parameter α is the deficit angle associated with conical geometry obeying $\alpha = 1 - 4\eta$, and η is the linear mass density of the string in natural unite $\hbar = c = 1$.

The DKP equation in curved space-time is given by [50–53]

$$\left[i\tilde{\beta}^\mu \left(\partial_\mu + \frac{1}{2}\omega_{\mu ab}S^{ab} \right) - m \right] \psi = 0, \quad (2)$$

where $\tilde{\beta}^\mu$ are the DKP matrices in curved space, and they satisfy the following relations:

$$\beta^a \beta^b \beta^c + \beta^c \beta^b \beta^a = \beta^a \eta^{bc} + \beta^c \eta^{ba}. \quad (3)$$

These matrices are related to the flat space-time β^a as: $\tilde{\beta}^\mu = e_a^\mu \beta^a$ with the following tetrad relations: The β^a are 5×5 matrices in the spin-0 representation and 10×10 matrices in the spin-1 representation. In our case, the β^a matrices are chosen as follows:[54, 55]

- for the spin-0 representation

$$\beta^0 = \begin{pmatrix} \theta_{2 \times 2} & 0_{3 \times 3} \\ 0_{3 \times 2} & 0_{2 \times 3} \end{pmatrix}, \quad \beta^i = \begin{pmatrix} 0_{2 \times 2} & \rho_{2 \times 3}^i \\ -\rho_{3 \times 2}^{iT} & 0_{3 \times 3} \end{pmatrix}, \quad (i = 1, 2), \quad (4)$$

with

$$\theta_{2 \times 2} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (5)$$

- for Spin-1 representation

$$\beta^0 = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 1}^+ \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1}^+ \\ -I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1}^+ \\ 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} & 0 \end{bmatrix}, \quad \beta^k = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & iK^{k+} \\ 0_{3 \times 3} & 0_{3 \times 3} & S_{3 \times 3}^k & 0_{3 \times 1}^+ \\ -I_{3 \times 3} & -S_{3 \times 3}^k & 0_{3 \times 3} & 0_{3 \times 1}^+ \\ iK^k & 0_{1 \times 3} & 0_{1 \times 3} & 0 \end{bmatrix} \quad (k = 1, 2), \quad (6)$$

where

$$0_{3 \times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$S^1 = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, S^2 = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad (7)$$

with

$$, K^1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, K^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix},$$

and $S^{ab} = [\beta^a, \beta^b]$. The spin connection $\omega_{\mu ab}$ obeys the relations

$$\omega_{\mu ab} = e_{al} e_b^j \Gamma_{j\mu}^l - e_b^j \partial_\mu e_{aj}, \quad \omega_{\mu ab} = -\omega_{\mu ba}, \quad (8)$$

with

$$\Gamma_{j\mu}^l = \frac{1}{2} g^{\lambda\lambda} (\partial_l g_{\mu\lambda} + \partial_\mu g_{\lambda j} - \partial_\lambda g_{\mu j}), \quad (9)$$

are the Christoffel symbols or the affine connections.

The introduction of the interaction will do by modifying the term $p_\mu \rightarrow p_\mu - qA_\mu$ where q is the electric charge, and $A_\mu = (-A_0, \vec{A})$ is the electromagnetic 4-vector potential. Takes into account the substitution $\partial_\rho \rightarrow \partial_\rho + m\omega\rho$ into Eq. (2), and the following form of the Coulomb potential [56–58]

$$qA^0 = \frac{f}{\rho} = \pm \frac{|f|}{\rho}, \quad (10)$$

where f is a constant, Eq.(2) can be written as

$$\left\{ i\tilde{\beta}^0 (\partial_0 - iqA_0) + i\tilde{\beta}^1 (\partial_\rho + m\omega\rho\varsigma) + i\tilde{\beta}^2 (\partial_\phi + \alpha [\beta^1, \beta^2]) - m \right\} \psi = 0, \quad (11)$$

with $\varsigma = 2(\beta^0)^2 - I$ and $\varsigma^2 = I$.

In what follow, this equation will be used to extract the eigensolutions of a both massive spin-0 and spin-1 particles.

III. THE EIGENSOLUTIONS OF A TWO-DIMENSIONAL DKP OSCILLATOR IN COSMIC STRING BACKGROUND

A. case of spin zero

The two dimensional DKP equation in cosmic string space-time subject to the Coulomb potential is given by Eq. (11), where m_0 is the mass of particles of spin-0. The stationary

state ψ is a five-component wave function of the DKP equation, which can be written as

$$\psi = \left(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5 \right)^T \quad (12)$$

Substituting (4), (5) and (12) into (11), we obtain

$$\left(E + \frac{f}{\rho} \right) \psi_2 - i \left(\partial_\rho - m_0 \omega \rho + \frac{1}{\rho} \right) \psi_3 - i \frac{\partial_\phi}{\alpha \rho} \psi_4 - m_0 \psi_1 = 0, \quad (13)$$

$$\left(E + \frac{f}{\rho} \right) \psi_1 - m_0 \psi_2 = 0, \quad (14)$$

$$i (\partial_\rho + m \omega \rho) \psi_1 - m_0 \psi_3 = 0, \quad (15)$$

$$i \frac{\partial_\phi}{\alpha \rho} \psi_1 - m_0 \psi_4 = 0, \quad (16)$$

$$- m_0 \psi_5 = 0. \quad (17)$$

From these equations, we get the following relations

$$\psi_2 = \frac{\left(E + \frac{f}{\rho} \right)}{m_0} \psi_1, \quad (18)$$

$$\psi_3 = i \frac{(\partial_\rho + m_0 \omega \rho)}{m_0} \psi_1, \quad (19)$$

$$\psi_4 = \frac{i}{m_0 \alpha \rho} \partial_\phi, \quad (20)$$

$$\psi_5 = 0. \quad (21)$$

Putting Eqs. (18), (19) and (20) into Eq. (13), we have

$$\left\{ \left(E + \frac{f}{\rho} \right)^2 + \left(\partial_\rho - m \omega \rho + \frac{1}{\rho} \right) (\partial_\rho + m \omega \rho) + \frac{1}{\alpha^2 \rho^2} \partial_\phi^2 - m_0^2 \right\} \psi_1 = 0. \quad (22)$$

Choosing as Ansatz $\psi_1 = e^{iJ\phi} \chi(\rho)$, and after simple algebraic manipulations, we arrive at

$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{\beta^2}{\rho^2} + \frac{2Ef}{\rho} - m_0^2 \omega^2 \rho^2 + \nu \right] \chi(\rho) = 0, \quad (23)$$

with

$$\nu = E^2 - m_0^2 + 2m_0 \omega, \quad \beta^2 = \lambda^2 - f^2, \quad \lambda = \frac{J}{\alpha}. \quad (24)$$

Now let us make a change of variable $\zeta = \sqrt{m_0 \omega} \rho$: in this case equation (23) becomes

$$\left[\frac{\partial^2}{\partial \zeta^2} + \frac{1}{\zeta} \frac{\partial}{\partial \zeta} - \frac{\beta^2}{\zeta^2} + \frac{\delta}{\zeta} - \zeta^2 + \frac{\nu}{m_0 \omega} \right] \chi(\zeta) = 0. \quad (25)$$

Taking the following substitutions [56–59]

$$\delta = \frac{2Ef}{\sqrt{m_0\omega}}, \chi(\xi) = e^{-\frac{\xi^2}{2}} \xi^{|\zeta|} H(\xi), \quad (26)$$

where we have write $\chi(\xi)$ as unknown function $H(\xi)$, Eq. (25) can be rewritten by

$$H''(\xi) + \left\{ (2|\beta| + 1) \frac{1}{\xi} - 2\xi \right\} H'(\xi) + \left\{ \frac{\nu}{m\omega} - 2|\beta| - 2 + \frac{\delta}{\xi} \right\} H(\xi) = 0. \quad (27)$$

The last equation is a biconfluent Heun function[57–59]

$$H(\xi) = H\left(2|\beta|, 0, \frac{\nu}{m\omega}, 2\delta, -\xi\right). \quad (28)$$

In order to solve Eq. (27), we use the Frobenius method [60–63]: Eq.(28) can be written as a power series expansion around the origin as

$$H(\xi) = \sum_{j=0}^{\infty} a_j \xi^j. \quad (29)$$

Substituting the series (29) into equation (27), we obtain the following recurrence relation:

$$a_{j+2} = -\frac{\delta}{(j+2)(j+1+\gamma)} a_{j+1} - \frac{\theta - 2j}{(j+2)(j+1+\gamma)} a_j, \quad (30)$$

where $\gamma = 2|\beta| + 1$ and $\theta = \frac{\nu}{m\omega} - 2|\beta| - 2$. By starting with $a_0 = 1$ and using the relation (30), we can calculate the other coefficients of the power series expansion as follows

$$a_1 = -\frac{\delta}{\gamma} = -\frac{2Ef}{\sqrt{m_0\omega}} \frac{1}{2|\beta| + 1}, \quad (31)$$

$$a_2 = \frac{\delta^2}{2\gamma(1+\gamma)} - \frac{\theta}{2(1+\gamma)} = \frac{2E^2 f^2}{m_0\omega} \frac{1}{(2|\beta| + 1)(2|\beta| + 2)} - \frac{\theta}{2(2|\beta| + 2)}. \quad (32)$$

Thus, the bound state solutions can be obtained by imposing the conditions where power series becomes a polynomial of degree n . This happens when:

$$\theta = 2n, a_{n+1} = 0. \quad (33)$$

with now $n = 1, 2, 3, \dots$. From Eq. (33), the eigenvalues are given by

$$E_{n,J}^2 = m_0^2 + 2m_0\omega_{n,J} \left(n + \left| \sqrt{\frac{J^2}{\alpha^2} - f^2} \right| \right). \quad (34)$$

Following Eq. (34), the spectrum of energy of the DKP oscillator subject to the Coulomb potential in the presence of the gravitational shows two interesting results: (i) firstly, all levels of energy are not degenerate, and (ii) secondly, the introduction of the Coulomb potential in the DKP oscillator modified the relativistic energy levels. This influence yields the ground state of DKP oscillator to be defined by the quantum number $n = 1$ in contrast to the quantum number $n = 0$. This situation is similar to the case of the Klein-Gordon oscillator studied by Bakke et al [36–39]. So we can extended their physical interpretation in our case. Thus, when the condition $a_{n+1} = 0$ imposed (Eq. (33)), we obtain a polynomial of degree n the power series expansion given in Eq. (29). So, we can assume that frequency $\omega_{n,J}$ of the DKP oscillator can be adjusted in order to satisfied the condition $a_{n+1} = 0$. As a consequence, the quantum number of the system n restrict the possible values of the angular frequency. According this, there are values of the angular frequency which are not allowed in the system. Now, let us examine the case of the ground state $n = 1$: the condition $a_{n+1} = 0$ yields $a_2 = 0$. So, by using Eq. (30), frequency $\omega_{n,J}$ is given by

$$\omega_{1,J} = \frac{2f^2 E_{1,J}^2}{m_0 (2|\beta| + 1)}, \quad (35)$$

which corresponds to the possible values of the angular frequency of the DKP oscillator in the ground state. The energy levels corresponding of this ground state are written by

$$E_{n,J}^2 = \pm \frac{m_0}{\sqrt{1 - \frac{4f^2}{2|\beta|+1} \left(1 + \left|\sqrt{\frac{J^2}{\alpha^2} - f^2}\right|\right)}}. \quad (36)$$

Therefore, the effects of the Coulomb potential on the spectrum of energy of the DKP oscillator in the presence of a cosmic string is given by a change of the energy levels, where the ground state is defined by the quantum number $n = 1$. Moreover, the values of the angular frequency of the DKP oscillator are restricted to a set of values in which allow us to obtain a polynomial solution to the biconfluent Heun series [36–39].

Finally, when we take the limit $f \rightarrow 0$ (i.e., vanishing of the Coulomb potential which here is chosen as time-like component of A^μ), we recover the exact result of scalar bosons in a cosmic string background [42]. Now, if we take a both limits $\alpha \rightarrow 1$ with $f \rightarrow 0$, we obtain the same result found in the case of a two-dimensional DKP oscillator in Minkowski space-time[54].

B. case of spin one

The two dimensional DKP equation in cosmic string space-time subject to a Coulomb potential is

$$\left\{ i\tilde{\beta}^0 (\partial_0 - iqA_0) + i\tilde{\beta}^1 (\partial_\rho + m\omega\rho\varsigma) + i\tilde{\beta}^2 (\partial_\phi + \alpha [\beta^1, \beta^2]) - M \right\} \psi = 0, \quad (37)$$

where M is the mass of spin-1 particles. The stationary state ψ is a ten-component wave function of the DKP equation with

$$\psi = \left(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9, \psi_{10} \right)^T. \quad (38)$$

Substituting Eqs. (6), (7) and (38) into Eq. (37), we found the following system of equations

$$- \left(E + \frac{f}{\rho} \right) \psi_7 + (\partial_\rho - M\omega\rho) \psi_{10} - M\psi_1 = 0, \quad (39)$$

$$- \left(E + \frac{f}{\rho} \right) \psi_8 + \frac{\partial_\phi}{\alpha\rho} \psi_{10} - M\psi_2 = 0, \quad (40)$$

$$- \left(E + \frac{f}{\rho} \right) \psi_9 - M\psi_3 = 0, \quad (41)$$

$$- \frac{\partial_\phi}{\alpha\rho} \psi_9 - M\psi_4 = 0, \quad (42)$$

$$(\partial_\rho + M\omega\rho) \psi_9 - M\psi_5 = 0, \quad (43)$$

$$\frac{\partial_\phi}{\alpha\rho} \psi_7 - \left(\partial_\rho + M\omega\rho + \frac{1}{\rho} \right) \psi_8 - M\psi_6 = 0, \quad (44)$$

$$- \left(E + \frac{f}{\rho} \right) \psi_1 + \frac{\partial_\phi}{\alpha\rho} \psi_6 - M\psi_7 = 0, \quad (45)$$

$$- \left(E + \frac{f}{\rho} \right) \psi_2 - (\partial_\rho - M\omega\rho) \psi_5 - M\psi_8 = 0, \quad (46)$$

$$- \left(E + \frac{f}{\rho} \right) \psi_3 + \left(\partial_\rho - M\omega\rho + \frac{1}{\rho} \right) \psi_5 - \frac{\partial_\phi}{\alpha\rho} \psi_4 - M\psi_9 = 0, \quad (47)$$

$$- \left(\partial_\rho + M\omega\rho - \frac{1}{\rho} \right) \psi_1 - \frac{\partial_\phi}{\alpha\rho} \psi_2 - M\psi_{10} = 0. \quad (48)$$

From Eqs. (41), (42) and (43), we have

$$\psi_3 = - \frac{\left(E + \frac{f}{\rho} \right)}{M} \psi_9, \quad (49)$$

$$\psi_4 = - \frac{1}{M} \frac{\partial_\phi}{\alpha\rho} \psi_9, \quad (50)$$

$$\psi_5 = \frac{1}{M} (\partial_\rho + M\omega\rho) \psi_9. \quad (51)$$

Putting these equations into Eq. (47), we arrive at the following equation for ψ_9

$$\left\{ \left(E + \frac{f}{\rho} \right)^2 + \left(\partial_\rho - M\omega\rho + \frac{1}{\rho} \right) (\partial_\rho + M\omega\rho) + \frac{\partial_\phi^2}{\alpha^2 \rho^2} - M^2 \right\} \psi_9 = 0 \quad (52)$$

For the other components, it very difficult to decouple the system of equations above as was done for ψ_9 . In order to overcome this problem, we use the same method as in Ref.[54]: if we choose

$$\psi_1 = \psi_2 = 0, \quad (53)$$

we obtain

$$\psi_6 = \psi_7 = \psi_8 = \psi_{10} = 0. \quad (54)$$

Now, Considering the following Ansatz for the component $\psi_9 = e^{iJ\phi} \varphi(\rho)$, we find

$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{\beta^2}{\rho^2} + \frac{2Ef}{\rho} - M^2 \omega^2 \rho^2 + \nu \right] \varphi(\rho) = 0, \quad (55)$$

with

$$\nu = E^2 - M^2 + 2M\omega, \quad \beta^2 = \lambda^2 - f^2, \quad \lambda = \frac{J}{\alpha}.$$

Eq (55) is similar to the Eq. (23) for the case of spin-0. Consequently, the eigensolutions are

$$E_n^2 = M^2 + 2M\omega_n \left(n + \left| \sqrt{\frac{J^2}{\alpha^2} - f^2} \right| \right), \quad (56)$$

$$\psi_9(\xi') = e^{-\frac{\xi'^2}{2}} \xi'^{|s|} H\left(2|\beta|, 0, \frac{\nu}{m\omega}, 2\delta', -\xi'\right), \quad (57)$$

with $\zeta' = \sqrt{M\omega}\rho$ and $\delta' = \frac{2Ef}{\sqrt{M\omega}}$.

As in the case of spin-0 particles, and from Eq. (56), the following remarks can be made: (i) the eigenvalues of particles of spin-1 have the same form that for the case of particles of spin-0, and (ii) all energy levels are not degenerate due the presence of the gravitational field of cosmic string, and finally (iii) the introduction of a Coulomb potential in DKP oscillator modified the relativistic energy levels. This influence yields the ground state of DKP oscillator to be defined by the quantum number $n=1$ in contrast to the quantum number $n=0$. In the limit where $\alpha \rightarrow 1$ and $f \rightarrow 0$, we find the same result as in the case of a two-dimensional DKP oscillator in Minkowski space-time [54].

IV. CONCLUSION

In this work, we have investigated the influence of the topological defects due to the cosmic strings space-time on the DKP oscillator subject to the scalar potential such as a Coulomb potential which is a time-like of the electromagnetic vector potential. The eigenvalues and eigenfunctions depend explicitly on the non local parameter of the space-time under consideration even though it is locally flat. Contrarily in the case of DKP oscillator for the flat space, the presence of the topological defects breaks the degeneracy of the spectrum of the DKP oscillator. In addition, we have seen that the presence of the Coulomb potential modifies the spectrum of energy of the DKP oscillator. In both cases, the ground state of the system is determined by the quantum number $n = 1$ instead of the quantum number $n = 0$. As consequently, the values of the angular frequency of the DKP oscillator, in both cases, are restricted to a set of values in which allow us to obtain a polynomial solution to the biconfluent Heun series.

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- [1] R. A. Krajcik et M. M. Nieto Am. J. Phys, **45**, 818 (1977).
N. Kemmer. Proc. R. Soc. Lond. A, 173, **91** (1939).
R. J. Duffin. Phys. Rev. **54**, 1114 (1938).
G. Petiau. Acad. R. Belg. Cl. Sci. M. Collect. 8, **16** (1936).
N. Kemmer, Proc. Cambridge Philosophical Society **39**, 189 (1943).
E.M. Corson, *Introduction to Tensors, Spinors, and Relativistic Wave-Equations*, Printed in Great Britain by Blackie & Son, Ltd., Glasgow (1953).
- [2] V. Ya. Fainberg and B. M. Pimentel, Phys. Lett. A, **271**,16-25 (2000)
R. Casana, V.Ya. Fainberg, 1, , B.M. Pimentel, J.S. Valverde, Phys. Lett. A, 316, 33–43 (2003).
- [3] E. Friedman, G. Kaelbermann and C. J. Batty, Phys. Rev. C, **34**, 16 (2000).
- [4] B. C. Clark, S. Hama, G. R. Kälbermann, R. L. Mercer and L. Ray, Phys. Rev. Lett, **6**, 2181 (1985).
- [5] R. E. Kozak, B. C. Clark, S. Hama, V. K. Mishra, R. L. Mercer and L. Ray, Phys. Rev. C, **40**, 2181 (1989).
- [6] R. C. Barrett and Y. Nadjadi, Nucl. Phys. A, **585**, 311c (1995).

- [7] S. Ait-Tahar, J. S. l-Khalili and Y. Nedjadi, Nucl. Phys. A, **589**, 307 (1995).
- [8] V. Gribov, Eur.Phys.J.C, **10**, 71 (1999).
- [9] I. V. Kanatchikov, Rep. Math. Phys. **46**, 107, (2000).
- [10] J.T.Lunardi, B.M.Pimental, R. G. Teixeira, J. S. Valverde, Phys. Lett. A, **268**,165-173 (2000).
- [11] J.T.Lunardi, B.M.Pimental, R. G. Teixeira, J. S. Valverde, Int. J. Mod. Phys. A, **17**,205 (2000).
- [12] M. DE Montigny, F. C. Khanna, A. E. Santana, E. S. Santos et J. D. M Vianna, J. Phys, **33**, L273 (2000).
- [13] L. K. Kerr, B.C. Clark, S. Hama, L. Ray et G. W. Hoffmann, Prog. Theor. Phys. **103**, 321 (2000).
- [14] A. Boumali, Can. J. Phys, **82**, 67–74 (2004).
- [15] A. Boumali, Can. J. Phys, **85**, 1417–29 (2007).
- [16] A. Boumali and L. Chetouani, Phys. Lett. A, **346**, 261 (2005).
- [17] A. Boumali, Phys. Scr, **76**, 669–73 (2007).
- [18] A. Okninski, Int. J. Theor. Phys, **50**, 729–736 (2011).
- [19] M. C. B. Fernandes and J. D. M. Vianna, Braz. J. Phys. **28**, 2 (1999).
- [20] M. C. B Fernandes, A.E. Santana and J. D. M. Vianna, J. Phys. A: Math. Gen. **36**, 3841 (2003).
- [21] V. Ya. Fainberg and B.M. Pimentel, Phys. Lett. **271A**, 16 (2000).
- [22] J. T. Lunardi, B. M. Pimentel, R.G. Teixeiri and J.S. Valverde, Phys. Lett. **268A**, 165 (2000).
- [23] W. B. Zeleny, Phys. Rev. **158**, 1223 (1967).
- [24] O. A. S. Valenzuela and R. E. Z. Vega, J. Phys. A: Math. Gen. **26**, 4967 (1993).
- [25] B. Boutabia and T. Boudjedaa, Phys. Lett. **338A**, 97 (2005).
- [26] L. Chetouani, M. Merad, T. Boudjedaa, and A. Lecheheb, Int. J. Theor. Phys. **43**, 1147 (2004).
- [27] P. Ghose, M. K. Samal and A. Datta. Phys. Lett. **315A**, 23 (2003).
- [28] J. T. Lunardi, B. M. Pimentel, and R. G. Teixeira, Gen. Relativ. Gravitation. **34**, 491 (2002).
- [29] R. Casana, B. M. Pimentel, J. T. Lunardi, and R. G. Teixeira, Gen. Relativ. Gravitation. **34**, 1941 (2002)
- [30] M. Nowakowski, Phys. Lett. A 244, 329 (1998)
- [31] L.B. Castro, A.S de Castro, Phys. Rev. A 90, 022101 (2014).
- [32] D. Itô, K. Mori and E. Carriere, Nuovo Cimento. A,**51**, 1119 (1967);

- M. Moshinsky and A. Szczepaniak, J. Phys. A: Math. Gen. **22**, L817 (1989).
- [33] A. Boumali, J. Phys. A: Math. Theor. **42**, 235301 (2009).
- [34] K. Bakke and C. Furtado, Ann. Phys. **355**, 48-54 (2015)
- [35] A. J. Carvalho, C. Furtado and F. Moraes, Phys. Rev. A. **84**, 032109 (2011).
- [36] K. Bakke, Eur. Phys. J. Plus. **127**, 82 (2012)
- [37] K. Bakke and F. Moraes, Phys. Lett. **A376**, 2838-2841 (2012)
- [38] K. Bakke and H. Belich, Eur. Phys. J. plus. **129**: 147 (2014)
- [39] K. Bakke, Ann. Phys. **341**, 86-93 (2014).
- [40] K. Bakke and C. Furtado, Ann. Phys. **355**, 48-54 (2015).
- [41] L. B. Castro, Eur. Phys. J. C. **75**, 287 (2015).
- [42] A. Boumali and N. Messai, Can. J. Phys, (2014)
- A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge University Press, (2000).
- [44] E. R. F. Medeiros, E.R. Bezerra de Mello, Eur. Phys. J. C 72:2051 (2012).
- [45] H. G. Dosch, J. H. Jansen, V. F. Müller, Phys. Nor. 5, 2 (1971)
- G. Soff, B. Müller, J. Rafelski, W. Greiner, Z. Naturforsch. A, J. Phys. Sci. 28, 1389 (1973).
- [46] B. Bergerhoff and G. Soff, Z. Naturforsch. **49** a, 997-1012 (1994).
- [48] R. L. L. Vitória, C. Furtado and K. Bakke, Ann. Phys **370**, 128-136 (2016).
- [49] R. L. L. Vitória and K. Bakke, Eur. Phys. J. Plus (2016) 131 :36.
- [50] M. Falek and M. Merad, Cent. Euro. J. Phys. **8**, 3 (2010)
- [51] K. Sogut and A. Havare, Class. Quantum Grav. **23**, 7129-7142 (2006).
- J. T. Lunardi, B. M. Pimentel, R. G. Teixeira, Duffin-Kemmer-Petiau equation in Riemannian space-times, arXiv:gr-qc/9909033
- [53] A.A. Bytsenko, A.E. Golcalves and B.M. Pimentel, World Scientific, pp 111, (2000).
- [54] A. Boumali, L. Chetouani and H. Hassanabadi, Can. J. Phys. **91**, 1-11 (2013)

- [55] W. Greiner, Relativistic Quantum Mechanics: Wave Equations, 3rd Edition (Springer, Berlin, 2000).
- [56] R. L. L. Vitória, C. Furtado, K. Bakke, Ann. Phys. **370**, 128 (2016)
- [57] E. R. Figueiredo Medeiros and E. R. Bezerra de Mello, Eur. Phys. J. C. **72**, 2051 (2012).
- [58] A. Vercin, Phys. Lett. B. **260**, 120 (1991).
- [59] J. Myrheim, E. Halvorsen and A. Verçin, Phys. Lett. B. **278**, 171 (1992).
- [60] S. Slavyanov and W. Lay, Special Functions: A Unified Theory Based on Singularities, Oxford University Press, Oxford, 2000
- [61] G. B. Arfken, H. J. Weber, Mathematical Methods for Physicists, sixth ed., Elsevier Academic Press, New York, 2005.

- P. Maroni, Sur la forme bi-confluente de l'equation de Heun, Comptes rendus de l'Académie des Sciences de Paris 264, 503-505 (1967).

- [63] A. Ronveaux, Heun's Differential Equations Oxford University Press, New York (1995).